SCENARIOS OF NONLINEAR WAVE TRANSFORMATION IN COASTAL ZONE

Yana Saprykina[1], Sergey Kuznetsov[1], Nataliya Andreeva[2], Margarita Shtremel[3]

Abstract

Based on field experiments and numerical modelling it is shown that depending on the average bottom slope and numbers of Iribarren and Ursell, the coastal zones could be classified according to manifestations of wind wave non-linearity, which herein is recognised as periodic energy exchange between first and second non-linear wave harmonics. The results could serve as a basis for development of vulnerability criteria of the coastal zone, taking into account its non-linear dynamics.

Key words: Iribarren number, Ursell number, nonlinear transformation of waves, parameterization of nonlinear waves, coastal zone vulnerability.

1. Introduction

Surface waves are the main source of energy in the coastal zone and largely determine its dynamics. Therefore, correct assignment of wave parameters and knowledge of how they change, when waves approach the shore, are needed to address all practical and theoretical problems of the coastal zone.

Transformation of waves in the coastal zone is a non-linear process enhanced by depth decrease. In general, waves in the coastal zone are weakly non-linear and dispersive, and their evolution is related to the growth of multiple non-linear harmonics. The latter results from near-resonant three-wave interactions, when the resonance condition is satisfied either for frequencies of non-linear harmonics or for wave vectors (for example, Bretherton, 1964):

$$\begin{aligned} &\pm k_1 \pm k_2 \mp k_3 = \delta_k, \\ &\pm \omega_1 \pm \omega_2 \mp \omega_3 = \delta_\omega \end{aligned} \tag{1}$$

where $k = k(\omega)$ is the wave number, determined by the dispersion relation and ω is the angular frequency. Approximate boundaries of the relative depths, when these conditions are satisfied, are $0.005 < h/L < 0.5$, where h is depth and L is wave length.

The main property of near-resonant interactions is periodic exchange of energy between the primary (or first) harmonic and the higher non-linear harmonics. Typical evolution of wave spectrum with a strong exchange of energy between first and second harmonics is shown in Fig. 1. Periodic exchange of energy between harmonics is demonstrated by periodic oscillations of their amplitudes in opposite phase: amplitude maxima of the first harmonic at distances of 240 m and 80 m and a maximum amplitude of the second harmonic at 150 m. Change in the ratio of amplitudes of the first and the second harmonics leads to fluctuations of the higher statistical moments of waves and affects the processes of sediment transport. For example, it is speculated that periodic in space exchange of energy between first and second harmonics could lead to formation of underwater bars (Boczar-Karakiewicz and Davidson-Arnott, 1987). Growth of higher non-linear harmonics also changes the symmetry of waves and, consequently, leads to asymmetry of wave velocity, which affects the sediment flux (Antsyferov et al., 2005). Periodic exchange of energy between harmonics leads to formation and disappearance of secondary waves, which significantly change the average wave period (Madsen and Sørensen, 1993). In this case, the apparent height of the secondary

[1]P.P.Shirshov Institute of Oceanology, RAS, Nakhimovskii pr., 36, Moscow, 117997, Russia. saprykina@ocean.ru, kuznetsov@ocean.ru

[2]F.Nansen Institute of Oceanology, BAS, ul. Pervyi may, 40, Varna 9000, Bulgaria. n.andreeva@io-bas.bg

[3]Moscow State University, Vorobievy gory, 1, 119991, Moscow, Russia. 6805919@mail.ru

waves is dependent on the amplitude of the second harmonic. Additionally, when individual waves in the coastal zone approach the shore, each individual wave has its own amplitude-frequency content, which is different from that of other waves. So, changes in parameters of the individual waves during this approach are also due to the periodic energy exchange (Saprykina et al., 2008).

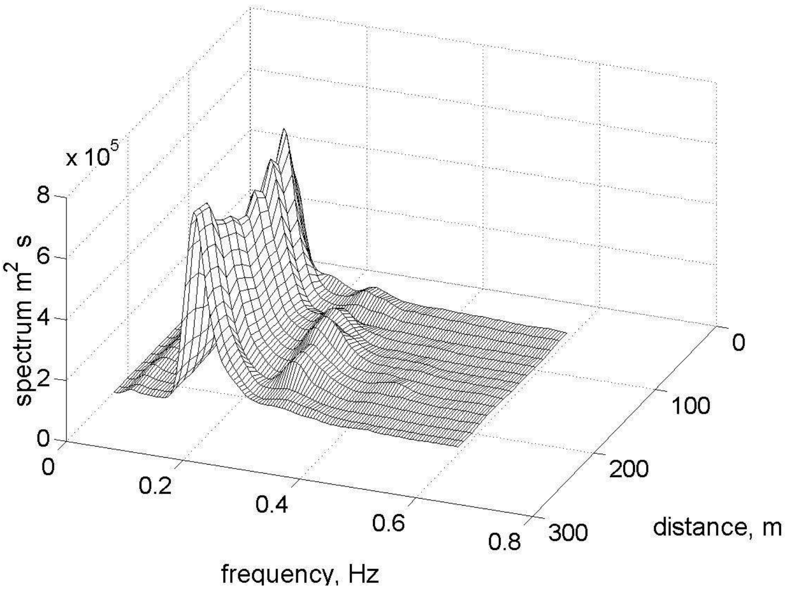

Figure 1. Spatial evolution of spectrum of irregular waves, showing a periodic energy exchange between first and second harmonics. Significant wave height at distance 230 m from the shore is 1.21 m. Wave spectra are normalized to the corresponding dispersions of wave ordinates. Wave data set No 39, registered on 29.09.2007 during field experiment "Shkorpilovtsy–2007"

Meanwhile, many coastal engineering calculations widely use the linear wave theory, which does not take into account the non-linear properties of waves, or the theory of shallow water waves, which assumes non-dispersion propagation of waves in the coastal zone, which corresponds to $\delta = 0$ in Eq. 1. In addition, the selection of these methods and disregard of wave non-linearity often has no clear justification. What values could reach the amplitudes of second harmonics, how they change in the coastal zone and whether one could predict their typical behavior for a particular coastal zone is still unknown. Neglect of non-linear effects in the coastal zone prevents full description of wave fields and does not allow establishing the exact mechanisms of sediment transport and bottom relief deformations.
The study aims, on the basis of wave parameters at the coastal zone entry and parameters of the coastal zone itself to construct a classification of coastal areas according to expected non-linearity of waves, and to develop criteria of realization of different wave transformation scenarios.

2. Basic parameters and experimental data

In general, main wave parameters are wave height and wave length. Since waves in nature are irregular, then as a characteristic wave height we will use the significant wave height H_s determined from wave spectrum as:

$$H_{sig} = 4\sqrt{m_0} \quad , \qquad m_0 = \int_0^\infty S(\omega)\, d\omega \tag{2}$$

$S(\omega)$ – wave spectrum and ω - angular frequency.
As characteristic wavelength a wave length corresponding to spectrum peak frequency is assumed. Degree of wave non-linearity, at the entry to the coastal zone and inside it, is determined by the following

dimensionless parameters: wave steepness *H/L* and relative wave height *H/h*. As a primary parameter of coastal zone bottom relief the average bottom slope α was taken. Furthermore, parameters relating the change of depth (or change in the average bottom slope) and the non-linear wave parameters are, on one hand, the number of Iribarren:

$$Ir = \frac{\tan\alpha}{\sqrt{H_0 / L_0}} \tag{3}$$

where α - average bottom slope, H_0– characteristic wave height, L_0– characteristic wave length; and on the other hand, the number of Ursell:

$$Ur = (2\pi)^2 [\bar{a}\, k \;/ (k\, h)^3] = \frac{1}{2}\frac{H}{h}\frac{L^2}{h^2} \tag{4}$$

where *a* is wave amplitude, *k* – wave number, *h* – water depth, *L* – wavelength, *H* – wave height.
Iribarren number gives the relation between the bottom slope ('steepness') of the coastal zone and the steepness of waves that are transformed in that coastal zone, while the number of Ursell – the relation between dispersive and non-linear properties of waves at a given water depth.
For analysis of wave transformation scenarios (change of amplitudes of the first and the second non-linear harmonics) and classification of coastal zones, based on the parameters described above, archive wave data were used. The data were collected during a field experiment that took place in September - October 2007 at the Scientific Research Base “Shkorpilovtsy” located on the Bulgarian Black Sea coast. Wave data were registered by 15 wire string wave gauges. Location of the wave gauges as well as bottom profile, are shown in Fig. 2. The mean bottom slope during the field experiment varied very slightly and was approximately equal to 0.022. Wave measurements in all 15 points were made simultaneously with a sampling frequency of 5 Hz and time duration of data series - 1 hour. The total number of wave record sets collected during the experiment was 65.

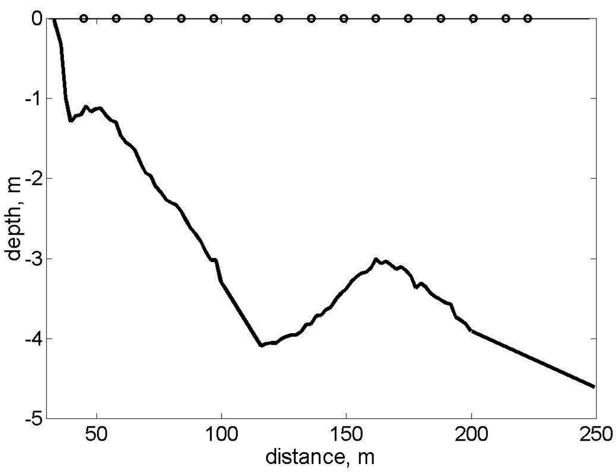

Figure 2. Bottom profile and location of wave gauges during the field experiment “Shkorpilovtsi–2007”.

3. Typical scenarios of wave transformation and criteria for their occurrence

Of all time series 23 wave data sets were selected, in which wave spectrum was narrow enough to difine frequency bands of first and second harmonics. The frequency bands were determined by the minimum between two peaks in the wave spectrum corresponding to peak frequencies of the first and the second harmonics. For example, in Fig. 1 the frequency bands of the first and the second harmonics are equal to

0.12-0.22 Hz and 0.22-0.33 Hz, respectively. Spatial energy evolution of frequency bands of the first and the second harmonics amplitudes (further in the text referred only as first and second harmonics) is presented as percentage of energy change of the corresponding frequency band, provided that the total energy of the wave spectrum over the entire frequency range is 100% (Fig. 3).

Analysis of the selected wave records showed that periodic exchange of energy between harmonics in the coastal zone, to a greater or lesser degree of intensity is always present, which made it possible to allocate four scenarios of second harmonic evolution (Fig. 3a-d).

1) at the entry to the coastal zone (or on the last seaward gauge) second harmonics are small, and their amplitudes grow only near the shore. That being the case 1.5–2 periods of energy exchange between first and second harmonics are detected (Fig. 3a).

2) within the coastal zone, only one distinct and complete period of energy exchange between the harmonics is observed and amplitude of the second harmonic peaks (Fig. 3b).

3) amplitude of the second harmonic has no pronounced maximum and slightly varies throughout the coastal zone. In this case, 3 or more periods of energy exchange are detected (Fig. 3c);

4) at the entry to the coastal zone the second harmonic has relatively large amplitude, which gradually reduces approaching the shore, and only about a half period of energy exchange between harmonics occurs (Fig. 3d).

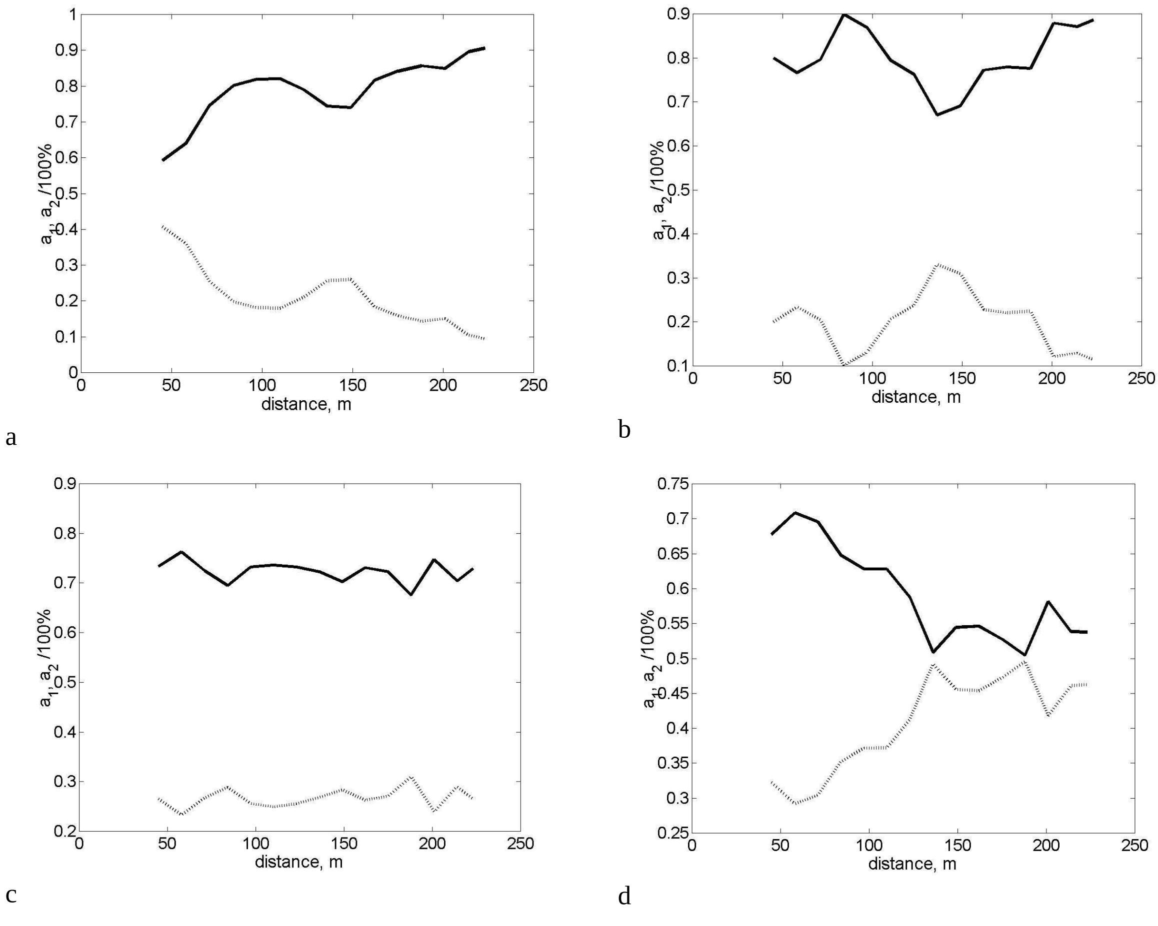

Figure 3. Scenarios of evolution of first and second harmonics in the coastal zone. Field experiment "Shkorpilovtsi–2007"; thick line - first harmonic amplitude (a_1), dotted line - second harmonic amplitude (a_2).

To verify that spectrum peaks detected on the second harmonics frequencies result from three-wave interactions a bi-spectral analysis was conducted. The presence of bi-coherence values ranging between 0.2 - 0.7 at frequencies of the main spectrum peak shows that the additional peaks at double the peak frequency

of the spectrum are really caused by second non-linear harmonics of the wave motion.
The dependence of numbers of Iribarren and Ursell on the parameters of wave non-linearity (H/L and H/h) for the four selected wave transformation scenarios are shown in Figs. 4-5, respectively. The calculations were made with wave data measured by the last wave gauge at a distance of 230 m from the shore and depth of 4.65 m.

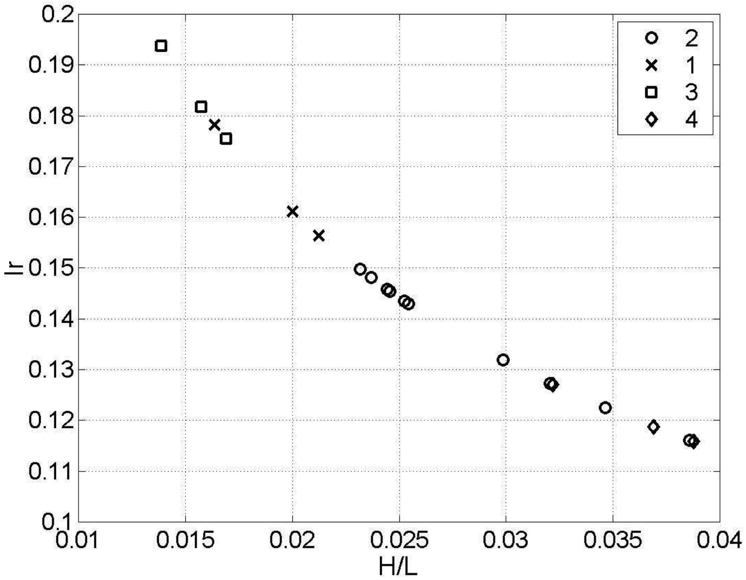

Figure 4. Disposition of the four wave transformation scenarios depending on the number of Iribarren and wave steepness in deep water. Field experiment "Shkorpilovtsi–2007".

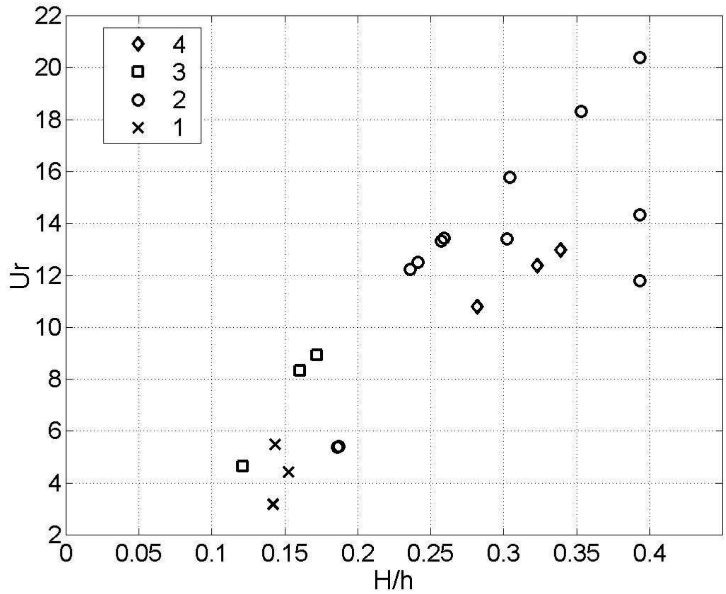

Figure 5. Disposition of the four wave transformation scenarios depending on the number of Ursell and relative wave height *H/h* in deep water. Field experiment "Shkorpilovtsi–2007".

In Fig. 4 it is clearly shown that the selected wave transformation scenarios and second harmonics evolution correspond to different ranges of Iribarren number. The most interesting, in terms of non-linearity manifestation, are scenarios 2 and 4, for which second harmonics have high or maximum amplitude within the coastal zone, and correspond to waves with steepness more than 0.022 and values of Iribarren number less than 0.15, i.e. those are steep waves transforming over mild sloped bottom. If wave steepness at the coastal zone entry is less than 0.022 and values of Iribarren number are more than 0.15, the amplitudes of second harmonics vary slightly or grow significantly only near the shore (scenarios 1 and 3). In this case, there are more than two periods of second harmonics variation (Fig. 3). For these scenarios, the number of Ursell is less than 10 and the relative wave height (shallow water non-linearity parameter) is less than 0.23 (Fig. 5). For waves with Ursell number more than 10 and relative wave height over 0.23 (Fig. 5), amplitudes of second harmonics are large, even at the coastal zone entry, or reach maximum values within it (scenarios 2 and 4) and there is one or a half period of second harmonics variation (Fig. 3).

Dependence of Ursell number on relative water depth (h/L) for the selected wave transformation scenarios is presented in Fig. 6. It could be stated that relative water depth, in this case varying from 0.095 to 0.15, which corresponds to transformation conditions of weakly non-linear dispersive waves, is not an indicative parameter for evolution of second harmonics. Moreover, for relative water depth, ranging from 0.095 to 0.115, scenarios 1, 3 and 4 could occur. At the same time, if Ursell number is more than 10, then scenarios, for which second harmonics grow high within the coastal zone (e.g. 2 and 4), come to pass. Thus, a combination of Ursell number equal to 10, along with Iribarren number less than 0.15 (wave steepness more than 0.022) could be used as a criterion to distinguish between different wave events. On one hand, these are events, for which second harmonics are high within the coastal zone (0.5-1 periods of energy exchange) and on the other hand, events, for which they grow approaching the shore or slightly vary throughout the entire coastal zone (2 or more periods of energy exchange).

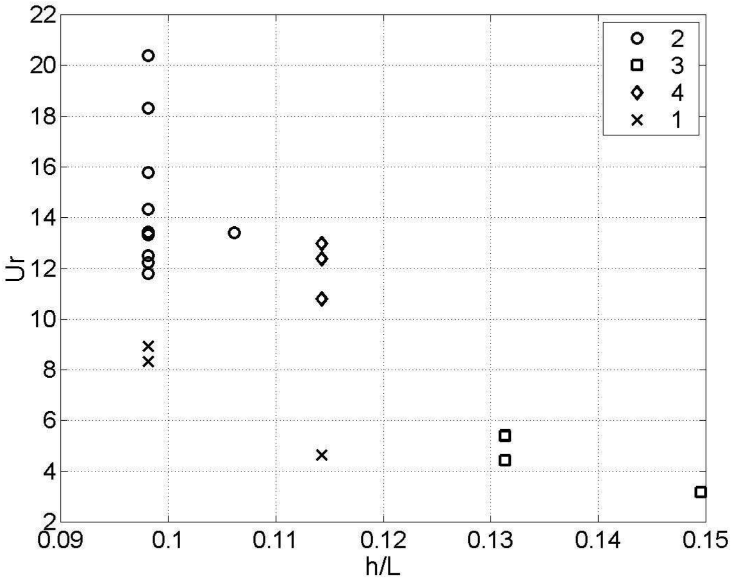

Figure 6. Disposition of the four wave transformation scenarios depending on the number of Ursell and relative water depth *h/L*. Field experiment “Shkorpilovtsi–2007”.

Unfortunately, on the basis of the experimental data it was not possible to establish a clear dependence or qualitative criterion for the magnitude of second harmonics neither from local numbers of Iribarren and Ursell, nor from their values at the entry to the coastal zone.

However, as shown above, the global numbers of Ursell and Iribarren together with the parameters of wave non-linearity, all calculated at the entry to the coastal zone, could be used to evaluate possible scenarios of transformation of the first and the second harmonics, and to classify coastal zones according to expected manifestations of non-linear properties of waves. Unfortunately, these parameters give no notion of the magnitude of second harmonics amplitude. Also, it should be noted that the criterion developed for the number of Iribarren was obtained only for a single coastal zone with an average bottom slope of 0.022.

How it will change if slope changes is unknown. To answer these questions numerical modelling was implemented.

4. Effects of bottom slope on the evolution of second harmonic amplitude

To determine the possible relative magnitude of the amplitude of second harmonic and to test the criterion, obtained by means of Iribarren number, transformation of irregular waves over a flat bottom with different bottom slopes was simulated. As input were taken JONSWAP waves with peak spectrum period T_p = 7 and H_s = 0.5, 1 and 2 m. The γ parameter was assigned a value of 6, which provided a fairly narrow spectrum in order to facilitate determination of frequency bands of first and second harmonics. Modeling was based on numerical solution of Boussinesq type equations with improved dispersion characteristics, in spectral form, taking into account wave breaking (Madsen and Sorensen, 1993, Kirby and Kaihatu, 1996). Bottom profile used for modelling purposes was designed as follows: from 15 m to 5 m depth - flat bottom with a slope of 0.05, and from 5 m up to 2 m depth, for different simulations, the slope had values of 0.01, 0.02, 0.03, 0.04, 0.06, 0.08 and 0.1, respectively.
Dependence of amplitudes of first and second harmonics on depth during transformation of waves over different bottom slopes is shown in Fig. 7. It is clearly seen that the amplitude of the second harmonic is determined by the distance the waves travel over the same depth. Therefore, waves propagating over mild sloped bottom and having longer run, could develop second harmonics with sufficiently large amplitudes. For instance, for waves with H_s = 1 m, the portion of second harmonic's energy could increase up to 40% of the first one (Fig. 7). As shown by simulations with other initial wave heights, increasing (or decreasing) the significant wave height of the initial waves leads to proportional increase (or decrease) of amplitude of the second harmonic.

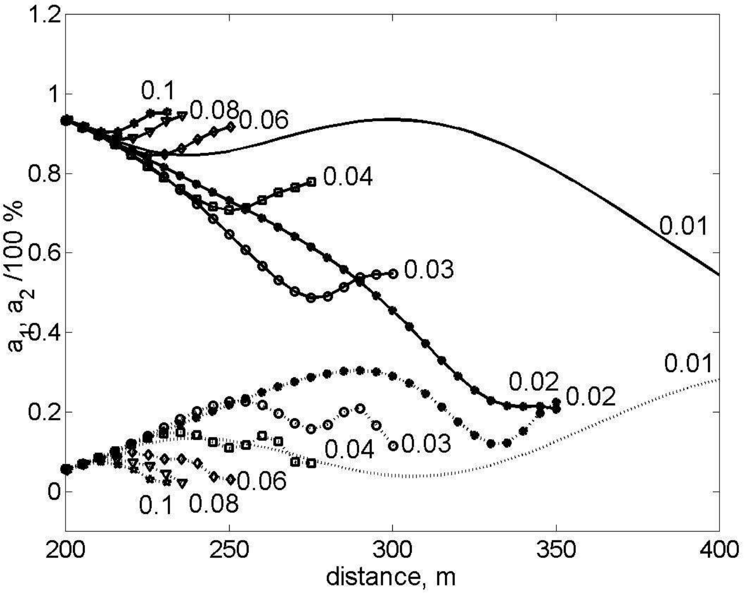

Figure 7. Evolution of energy of first and second harmonics during transformation of waves with JONSWAP spectrum and initial Hs = 1 m, Tp = 7 s, γ = 6, propagating over different bottom slopes; thick line - first harmonic amplitude (a_1), dotted line - second harmonic amplitude (a_2).

It should be noted that, starting with bottom slope equal to 0.06 and steeper slopes, the second harmonic grows insignificantly, reaching less than 10% of the first harmonic (Fig. 7). This is due to the fact that on steep shores wave breaking begins earlier, since the depth decreases more rapidly, and the process of breaking fully controls the growth of the second harmonic (Kuznetsov, Saprykina, 2004). Thus, the amplitude of the second harmonic depends on the bottom slope and it could be stated that with waves transforming over steep shores (bottom slope more than 0.06), this amplitude would be small.

Slow decrease of water depth and sufficiently long wave run most clearly demonstrate the dispersive and non-linear nature of waves, when periodic energy exchange between harmonics is present and amplitude of the second harmonic reaches its maximum and then decreases again. Thus, for bottom slopes of 0.01 two periods of periodic energy exchange could be observed, while for slopes equal to 0.03 - only one. Therefore, for waves transforming over mild sloped beaches with slopes less than 0.04, an increase in amplitudes of the second harmonics to their maximum values could be predicted, while for slopes less than 0.03 a periodic energy exchange between the first and the second harmonics could also be expected. On the other hand, for waves transforming over steep shores, with slope more than 0.04, the amplitude of the second harmonic grows insignificantly and the periodic energy exchange between the harmonics is not very well pronounced.

Dependence of Iribarren number on wave steepness for different bottom slopes for the four selected wave transformation scenarios is presented in Fig. 8. It can be seen that with bottom slope increase so does the steepness of waves, and they will have very well pronounced one period of energy exchange, corresponding to scenario 2 (Fig. 8) or scenario 4 - large second harmonic at the coastal zone entry. Furthermore, the more the bottom slope steepens the more the waves will transform according to scenarios 1 and 3, i.e. second harmonics will stay small throughout the coastal zone or grow only approaching the shore (Fig. 8).

A rough estimate of the simulated and experimental data allowed development of boundary values based on wave steepness and Iribarren number, within which the waves are transformed according to the presented scenarios. For Ir > 7*H/L (tgα > 7*$(H/L)^{3/2}$) wave transformation by scenarios 1 and 3 will come to pass, while for Ir < 7*H/L (tgα < 7*$(H/L)^{3/2}$) by scenarios 2 and 4.

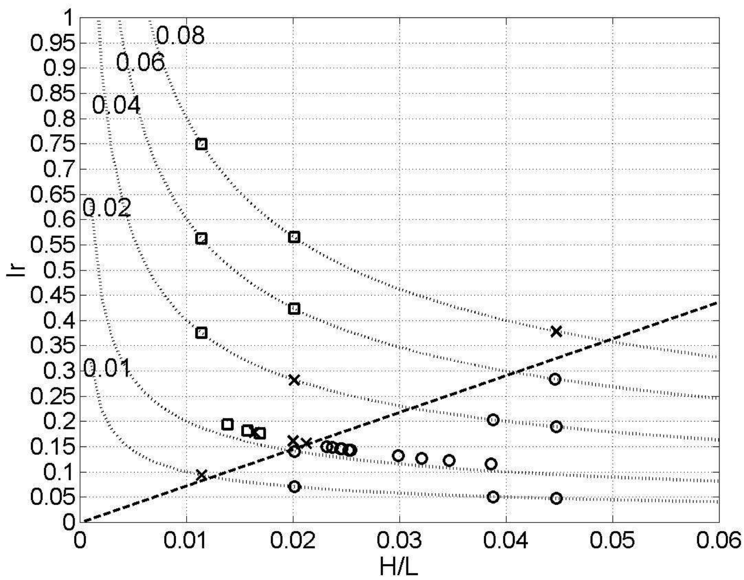

Figure 8. Disposition of the four wave transformation scenarios depending on the number of Iribarren (dotted curves) and wave steepness for different bottom slopes: 0.01, 0.02, 0.04, 0.06 and 0.08. Straight dashed line stands for Ir = 7 * H / L, delimiting wave transformation scenarios marked by crosses for Scenario 1, circles for Scenario 2 and squares for Scenario 3. Symbols outside the curves show experimental data, while symbols lying on the curves - the simulated data.

Dependence of the second harmonic value, obtained by modelling of wave transformation with different initial significant wave heights over different bottom slopes, on the Iribarren number at the coastal zone entry (depth of 5 m) is shown in Fig. 9. It is evident that the greater the initial wave height, the larger the amplitude of the second harmonic. However, when Iribarren number is more than 0.27 the amplitude of the second harmonic does not grow remaining relatively small: 27% of the wave energy (for Hs = 2 m) up to

9% (for Hs = 0.5 m).
For Iribarren numbers less than 0.27 the amplitude of the second harmonic can reach large relative values. Thus, at the coastal zone entry the number of Iribarren could be used as a criterion for expected relative values of the second harmonic amplitude within the coastal zone.

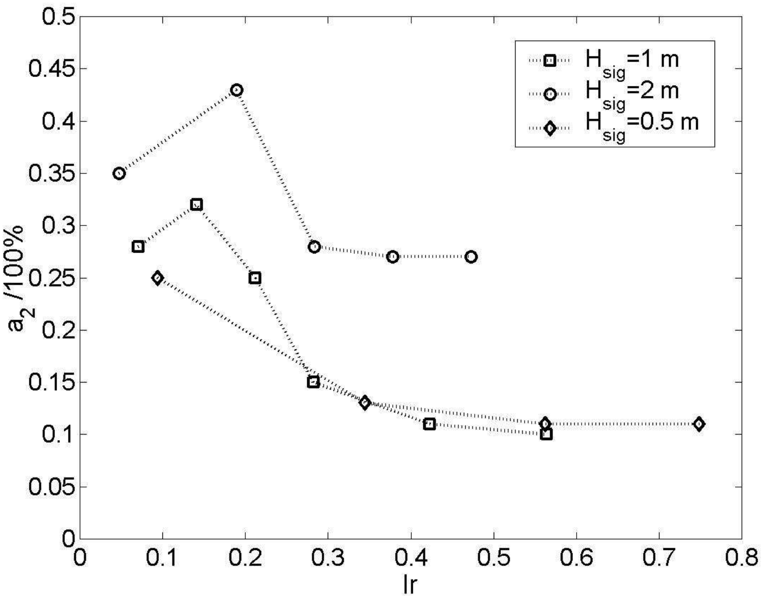

Figure 9. Dependence of second harmonic amplitude on Iribarren number for simulated wave with JONSWAP spectrum and initial parameters H_s = 0.5 m, 1 m, 2 m, T_p = 7 and γ = 6.

5. Conclusions

The study showed that depending on the average bottom slope and numbers of Iribarren and Ursell, coastal zones could be classified according to expected manifestations of wave non-linearity, herein recognised as periodic energy exchange between first and second harmonics of wind waves.
Second harmonics with large relative amplitudes and periodicity of energy exchange between the first and the second harmonics are common for shallow shores with bottom slope less than 0.03.
Ursell number (Ur) equal to 10, at the entry to the coastal zone, could be used as a criterion for differentiation between wave transformation scenarios. If Ur > 10, amplitudes of the second harmonics are large even at the coastal zone entry, or grow high due to the periodic energy exchange within the coastal zone, i.e., a complete or a half cycle of energy exchange is observed. For such waves Ir> 7 * H/L, where H/L is steepness of waves entering the coastal zone. If Ur < 10 and Ir <7 * H / L, then two or more periods of energy exchange could be detected and amplitudes of the second harmonics increase approaching the shore or remain small.
On the other hand, the number of Iribarren, at the entry to the coastal zone, could be used as a criterion for expected relative amplitudes of second harmonics within the coastal zone, i.e. if it is less than 0.27 than the relative amplitude of the second harmonic will be large. Thus, the above described coastal zones could be attributed to coastal zones, for which non-linearity of waves in terms of periodic energy exchange between the first and the second harmonics and growth of relatively large amplitude of the second harmonic, is the most expected. Therefore, the Black Sea coast, where the field experiment ‘Shkorpilovtsi-2007’ was performed, is one such coastal area.
The results, verified by a large number of experimental data, could be useful for development of criteria for vulnerability of coastal zones, taking into account their non-linear dynamics.

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