A MODEL FOR BEACH PLAN SHAPE CHANGE USING AN INVERSE APPROACH

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Abstract

The paper describes and demonstrates a 'reduced-physics' beach morphodynamic model based on an inverse modeling approach. The model, which is developed on a form of advection-diffusion equation, incorporates two site-specific parameters, sediment diffusion coefficient and a source function, to describe the effects of dynamic forcing on beach plan shape evolution. The sediment diffusion coefficient describes the beach plan shape change associated with the incident wave field while the source function describes all processes other than incident waves, including the effects of tides, wave induced currents and other anthropogenic processes. Those two parameters are determined by combining a data-driven approach with an inverse modeling technique. The development of the model, determination of the model unknowns and an analysis of the beach behavior through model application are demonstrated for Colwyn Bay Beach in North Wales, UK.

Key words: Coastal morphodynamics, shoreline change, cross-shore transport, longshore transport

1. Introduction

Coastal planning and management requires morphodynamic changes at a number of time scales. To determine short-term, local-scale coastal morphodynamic change, two or three dimensional hydrodynamic models coupled with sediment transport and bed updating modules are used (Van Rijn et al., 2003; Roelvink, 2001; and many others). These models are very useful tools for understanding and assessing local scale coastal dynamics at short time scales. Whilst attractive in terms of their physical principles and generic applicability, detailed process models are (a) computationally expensive; (b) dependent on having or creating suitable data for boundary conditions and, (c) prone to sensitivity to the specified initial conditions. They have inherent limitations when applied to large scale and longer term time scales, as a result of process driver uncertainties, great sensitivity to initial conditions and large computation times.

To overcome such limitations of process-based models, 'reduced physics' models were developed in the engineering literature (e.g. Pelnard-Considere 1956; de Vriend et al., 1993) and 'behaviour-oriented' models in the geomorphological literature, (e.g. Murray and Paola, 1994; Coulthard et al., 2002) to determine long term coastal morphodynamics. In this type of models some elements of the physics are retained in order to reduce computational costs and simplify the dynamics on the assumption that the broad scale morphological changes will be captured. They use simplified governing equations that exhibit the behaviour of the application. Good examples of reduced physics models in the current context are the 1- or N-line shoreline evolution models (Pelnard-Considere, 1956; Hanson and Kraus, 1989; Hanson et al., 2003; Dabees and Kamphius, 1998) and cross-shore profile evolution models of Stive and de Vriend (1995) and Niodoroda et al. (1995).

Another alternative is 'data-driven' models. The performance of such models is highly dependent on the quality of the observations and predictions will usually be restricted to conditions that fall within the range, defined by the observational dataset. Nevertheless, data-driven approaches have had some success in predicting medium to long term shoreline change when ample data are available at a site, (see e.g. Rozynski 2003; Reeve et al., 2008; Horrillo-Caraballo and Reeve, 2008, 2010).

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This paper describes the formulation of the model, combining a reduced physics formulation with a data-driven approach to describe beach plan shape evolution at medium-term timescales, i.e. of the order of years to a decade. The model is based on a form of advection-diffusion formulation. Section 2 of the paper describes model development. In Section 3, the field site used to demonstrate the method is described. Results are discussed in Section 4. Section 5 concludes the paper.

2. Model Development

The model builds upon the formulations used by Pelnard-Considere (1956) and Larson et al. (1997) for 1-line shoreline model. The governing equation used to describe beach plan shape evolution relative to a fixed line of reference is:

$$\frac{\partial y}{\partial t} = \frac{\partial}{\partial x}\left(K(x,t)\frac{\partial y}{\partial x} \right) + S(x,t) \qquad (1)$$

Equation (1) describes spatial and temporal variation of shoreline position $y(x,t)$ defined relative to a fixed reference line at a longshore location x. t is time. $K(x,t)$ is interpreted as the space- and time-dependent diffusion coefficient which relates the response of the shoreline to the incoming wave field. $S(x,t)$ is a space- and time-dependent source function which describes all processes that contribute to shoreline change other than incident waves. That includes tides, wave induced currents and other anthropogenic impacts. Therefore, $K(x,t)$ and $S(x,t)$ are the key parameters that govern the success of the model. The schematic of the model is shown in Figure 1.

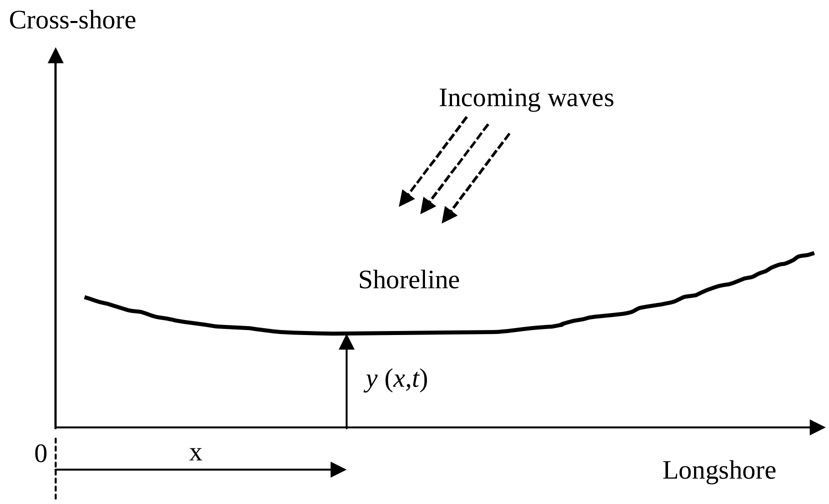

Figure 1 – Schematics of the beach plan shape model.

Both $K(x,t)$ and $S(x,t)$ are site specific unknowns that must be determined before applying the model to a given beach. If suitable parameterisations for these two unknowns can be found then, the model governing equation can be used to predict shoreline change by time-stepping the equation into the future. Here, the primary aim is the demonstrate a methodology for determining both $K(x,t)$ and $S(x,t)$ from observations.

Solving Equation (1) to find K and S simultaneously, given a sequence of observations of beach plan shape is a challenging mathematical problem. Here, we adopt an approximate, two stage method to parameterise $K(x,t)$ and $S(x,t)$. The procedure starts with Reynolds expansion of the model variables. Thus the variables are written as a summation of a time averaged component and a time-varying residual:

$$\left.\begin{aligned} y(x,t) &= \bar{y}(x) + y'(x,t) \\ K(x,t) &= \bar{K}(x) + K'(x,t) \\ S(x,t) &= \bar{S}(x) + S'(x,t) \end{aligned}\right\} \tag{2}$$

In Equation (2), over-bar denotes the time-averaged components and the prime denotes time varying residuals. Equation (1) can then be rewritten to give:

$$\frac{\partial\left[\bar{y}(x)+y'(x,t)\right]}{\partial t} = \frac{\partial}{\partial x}\left(\left[\bar{K}(x)+K'(x,t)\right]\frac{\partial\left[\bar{y}(x)+y'(x,t)\right]}{\partial x}\right) + \bar{S}(x) + S'(x,t) \tag{3}$$

If all terms with time-varying residual component of the diffusion coefficient were included in the unknown source function, Equation (3) can be written in terms of a new unknown source function $G(x,t)$ as in Equation (4).

$$\frac{\partial\left[y(x,t)\right]}{\partial t} = \frac{\partial}{\partial x}\left(\bar{K}(x)\frac{\partial y(x,t)}{\partial x}\right) + G(x,t) \tag{4}$$

where

$$G(x,t) = \frac{\partial}{\partial x}\left(K'(x,t)\frac{\partial\left[\bar{y}(x)+y'(x,t)\right]}{\partial x}\right) + \bar{S}(x) + S'(x,t) \tag{5}$$

In addition to the contributions from tidal and other morphodynamic drivers to beach change, the new source function $G(x,t)$ now contains the residual component of wave-induced stresses.

Equation (4) forms the modified governing equation for describing beach plan shape change in time and space. The physical interpretation of the Equation (4) is that the first term in the right hand side of Equation (4) determines the morphodynamic response of the shoreline to the time-averaged incident wave climate. This is mostly associated with longshore transport and hence shoreline change at time scales longer than days or seasons. The shoreline response to individual wave events, seasonal variation of incoming wave climate and other non-wave driven impacts are modelled by the second term. This may be associated with cross-shore transport and short term residual shoreline change.

The success of the model strongly depends on the ability to find suitable parameterisations for $\bar{K}(x)$ and $G(x,t)$. The methodology we propose here to determine $\bar{K}(x)$ and $G(x,t)$ involves historic measurements of incident wave climate and shoreline surveys, and is described below.

Our interest here is on beach changes taking place at seasonal to annual timescale. Therefore, as a first approximation, we assume that shoreline plan shape change is primarily governed by longshore transport. Based on this assumption, we can write the following expression for diffusion coefficient, following Larson et al. (1997) as in 1-line shoreline model:

$$K(x,t) = \frac{2Q_0}{D_c} \tag{6}$$

In equation (6), Q_0 is the amplitude of longshore sediment transport rate and D_c is the depth of closure. Here we use the CERC equation for longshore transport (Shore Protection Manual, 1984) to determine $Q_0(x,t)$.

$$Q_0 = \frac{\rho}{16}[H_b(x,t)]^2 c_{gb} \frac{K^*}{(\rho_s - \rho)\lambda} \tag{7}$$

where ρ is the density of seawater, ρ_s is the sediment density, K^* is empirical coastal coefficient, λ is porosity of the beach, c_{gb} is wave group velocity at wave breaking point and H_b is wave height at wave breaking. Q_0 is a function of H_b and therefore varies with time and space.

The depth of closure, D_c, can be determined form the incoming wave climate using the equation (Hallermeier, 1978)

$$D_c = 2.28 H_{s12} - 68.5 \frac{H_{s12}}{g T_s^2} \tag{8}$$

where H_{s12} is mean of the highest 1/12th wave heights and T_s is the associated wave period.

Once the time history of incident waves are known then Q_0 can be determined from Equation (7), which can then be used in Equation (6) to determine $K(x,t)$. The time mean diffusion coefficient $\overline{K}(x)$ can be obtained by averaging $K(x,t)$.

Once $\overline{K}(x)$ is determined following the procedure described above, a solution to Equation (4) is sought to recover the source function $G(x,t)$, using an inverse modeling technique. The inversion procedure relies on historic measurements of shoreline positions. The solution to Equation (4) can be given as follows.

For convenience, Equation (4) is re-written in operator notation to give

$$y_t = Dy + G \tag{9}$$

where D is the Laplacian operator $D = \frac{\partial}{\partial x}\left(\overline{K}(x)\frac{\partial y}{\partial x}\right)$ and y_t is the time derivative of y.

We assume that the time interval between two successive historic shoreline position measurements is τ and that time variation of $G(x,t)$ during that time interval is small enough to be neglected. Then, the formal discrete inverse solution of Equation (9) to the first order can be written as (Spivack and Reeve, 2000)

$$y(x_i, t_{j+1}) \cong (\exp(D\tau) - 1) D^{-1} G(x_i, t_{j+1/2}) + \exp(D\tau) y(x_i, t_j) \tag{10}$$

where t_j and t_{j+1} and, $y(x_i,t_j)$ and $y(x_i,t_{j+1})$ are the shoreline positions at the i^{th} longshore node at $(j)^{th}$ and $(j+1)^{th}$ time steps respectively. The exponential terms are differential operators acting on the functions $G(x,t)$ and $y(x,t)$. Using a first order approximation of exponential terms, an explicit expression for the source function $G(x,t)$ may be found from Equation (10):

$$G(x_i, t_{j+1/2}) = \frac{1}{\tau}\left[y(x_i, t_{i+1}) - \exp(D\tau) y(x_i + t_j)\right] \tag{11}$$

Given a time series of shoreline position $y(x,t)$ and time mean diffusion coefficient $\overline{K}(x)$, a discrete time series of the source function $G(x,t)$ can be recovered from Equation (11).

3. Study Site

We selected Colwyn Bay beach in North Wales, UK as the study site. Colwyn Bay forms a part of the Conwy Bay beach system in North Wales. The beach is bounded by two headlands, Rhos-on-Sea to the west and Penmaen Head to the east. The longshore movement of sand is largely confined between these two headlands. The beach plan shape exhibits significant changes in shape. During the last two decades, the beach has been extensively monitored by the Conwy County Borough Council, UK, as a part of a sustainable development and coastal management plan (Halcrow Maritime, 2000). As a result, it is rich in historic field measurements, including bathymetry and shoreline surveys, water level measurements and incident wave records. **Figure 2** shows the location of Colwyn Bay in the UK, an aerial view of the beach and a view looking towards Rhos-on-Sea.

Colwyn Bay comprises a sandy beach with a thin surface layer of coarse material. The beach is about 3 km long. The beach sand has a median grain diameter (D_{50}) varying between 0.2 and 0.4 mm. A part of the beach is fronted by a promenade. The gradients of the upper and lower beaches are about 1:10 and 1:60 respectively, (HR Wallingford, 1988). Colwyn Bay experiences semi-diurnal tidal fluctuations. The coast is macro-tidal, with the tidal range being 7.2m at spring tides and 3.5m at neap tides. Mean water level is 4.38m above chart datum. Mean high water spring (MHWS) is +3.88m and mean low water spring (MLSW) is -3.32 m relative to chart datum (CD), (Halcrow Maritime, 2000).

Colwyn Bay is exposed to waves arriving from the west to north-east. Deepwater wave conditions off the coast of Colwyn Bay have been derived by wave hindcasting using UK Meteorological Office wind data at Squires Gate, Liverpool, UK, from 1979 to 1987 (HR Wallingford, 1988). The offshore hindcast wave records have then been transformed to near-shore, using the HR Wallingford wave transformation model (HR Wallingford 1988). During a follow-up study, Conwy Council derived wave data at 5 near-shore locations along Colwyn Bay, using hindcasted offshore directional wave data from 1986 to 2006.

Figure 2 – (a) Map of the UK and the location of Colwyn Bay (b) Map of Colwyn Bay (Google Earth) and (c) View of the beach (http://www.google.co.uk/images).

Beach surveys are available between 2001 and 2005, at six monthly intervals. The surveyed data were compiled into bathymetry maps relative to Chart Datum and provided by Conwy County Borough Council, UK. It should be noted that the eastward beach segment of the Colwyn Bay was not covered in these surveys as it is maintained by other authorities. All bathymetry survey maps were digitised to determine the mean high water neap (MHWN) contour, which was taken as the 'shoreline'. All shoreline contours were then transformed to a new Cartesian coordinate system where *x*- and *y*-coordinates correspond to longshore and cross-shore directions respectively, as required by the model. The shorelines were then interpolated to uniform 20 m longshore intervals, using the Akima interpolation routine (Akima, 1969). This was necessary as the method used to recover the source function demands shoreline data at uniform longshore intervals. The mean shoreline and the shoreline envelope determined from measured beach surveys are shown in **Figure 3**.

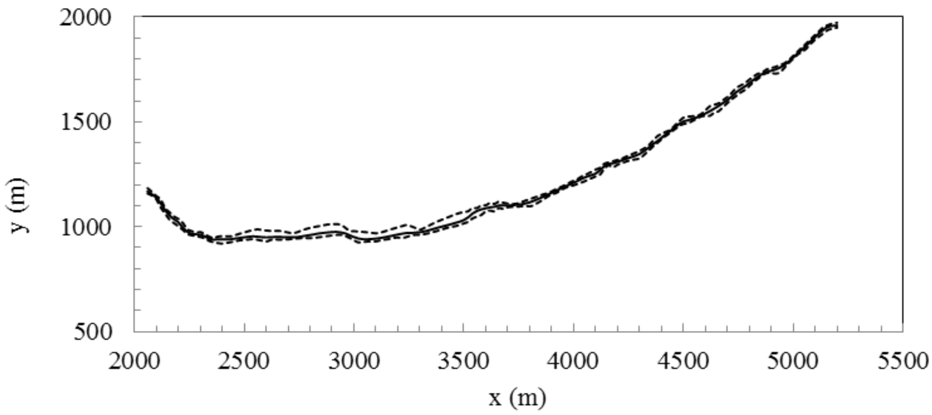

Figure 3 – The mean shoreline and the shoreline envelope at Colwyn Bay during 2001 and 2005.

4. Model Application

Both the time mean diffusion coefficient $\overline{K}(x)$ and the source function *G*(*x*,*t*) for Colwyn Bay were determined using the method described in Section 2 of the paper. First, time-varying values of Q_0 along the shoreline were determined from Equation 7. Then, Equation 6 was used to determine *K*(*x*,*t*). The mean diffusion coefficient $\overline{K}(x)$ was then calculated by time averaging *K*(*x*,*t*). Spatial variation of $\overline{K}(x)$ along Colwyne Bay beach is given in **Figure 4**.

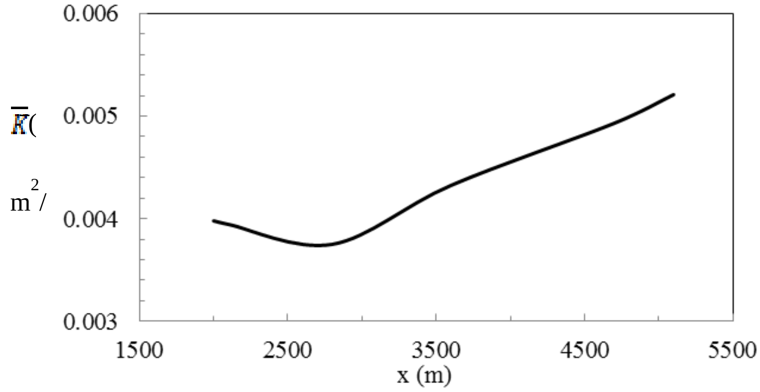

Figure 4 – Variation of the mean diffusion coefficient along the Colwyn Bay beach.

The time-mean diffusion coefficient represents the contribution of net effect of incident waves on shoreline plan shape change. The variation of the mean diffusion coefficient along the beach is an indication to longshore variability of the net wave effect on the beach plan shape change. The figure shows that mean diffusion coefficient is larger in the east side of the Colwyn Bay than the west side thus showing the dominance of longer term changes in the east than the west.

Once the mean diffusion coefficient is resolved from the incident wave data, the time and space-varying source function $G(x,t)$ was determined from the Equation 11. Equation 11 provides a discrete time series of $G(x,t)$, for a given discrete set of shoreline surveys. **Figure 5** (top panel) shows the spatial variation of the source function envelope. Maximum time variations of source function at each longshore location are shown in the bottom panel.

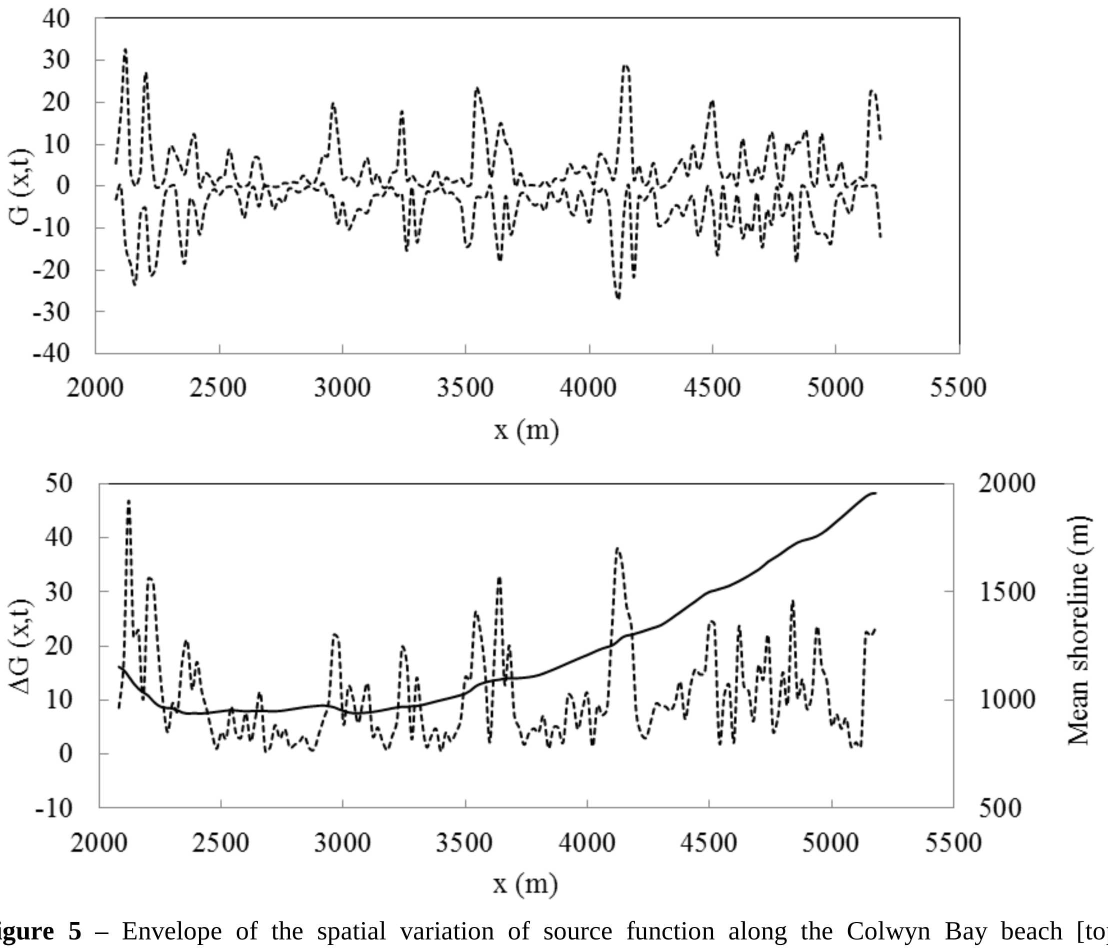

Figure 5 – Envelope of the spatial variation of source function along the Colwyn Bay beach [top] Maximum variation of the source function at all longshore locations (dotted line) and mean shoreline position (dark line) [bottom].

It is seen in this figure that the effects other than net wave effect on beach plan shape change is highly localised along the shoreline. The source function at the west side of the Colwyn Bay shows a significantly large variability than at the east side thus indicating higher short-term beach change in the west side. Localised large variations at the central part of the bay may be attributed to the presence of several groynes (Halcrow Maritime, 2000).

It should be noted that different morphodynamic drivers included in the source function act at different time scales. Fluctuating component of wave effects are at the shortest time scale. The effects of wave induced currents may work at a longer time scale as a results of seasonal variations of the incident wave field. Tidal and other anthropogenic effects such as sea defences may work at even longer time scale. To investigate the effects at different time scales, time variation of $G(x,t)$ at all longshore locations and its envelope (top panel) and incident waves measured during the corresponding period (bottom panel) are shown in Figure 6. In Figure 6 (top panel), each individual curve corresponds to the time variation of $G(x_i,t)$ for a fixed point x_i along the beach. Where the overall beach is concerned, a consistent seasonal fluctuation of $G(x,t)$ are clearly evident. A noticeable increases in $G(x,t)$ can be seen during winter months, which associate with high energy seas (see Figure 6 bottom panel). It is evident from this observation that the effect of residual incident waves dominates the source function.

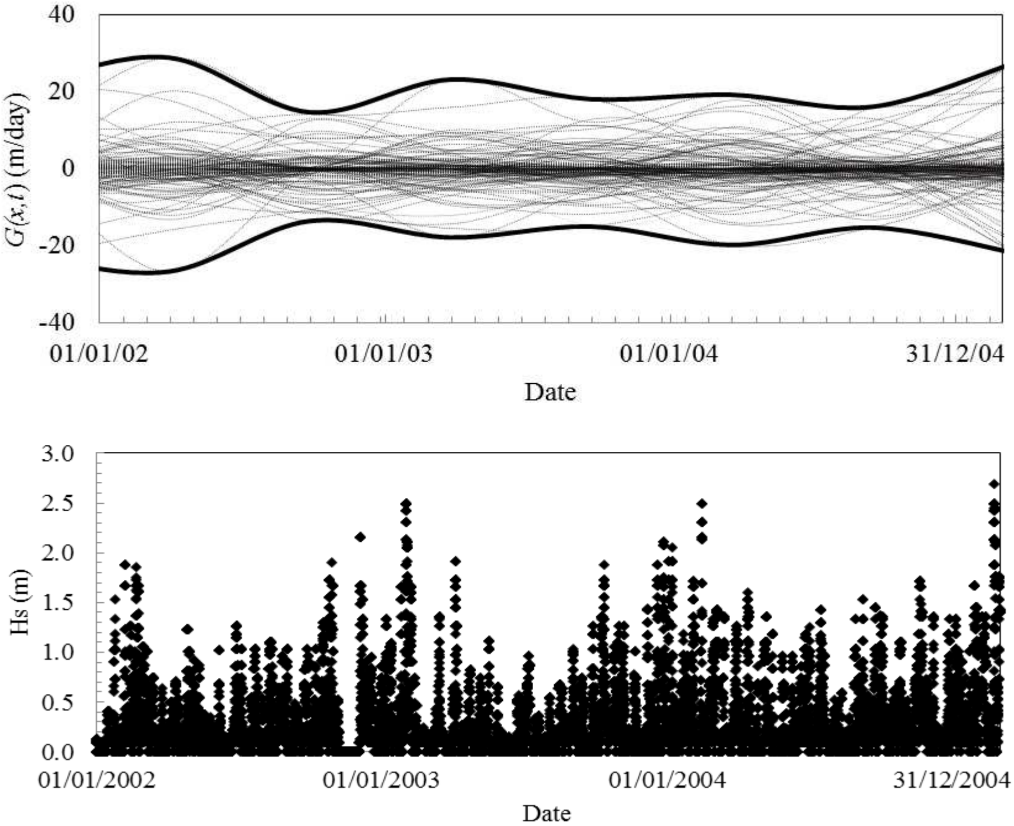

Figure 6 – Temporal variation of the source function [top] and the measured incident waves during the corresponding time period [bottom].

The source functions at each time interval were then spatially integrated to determine the overall effect of all contributions other than net effect of incident waves on beach change. The space integrated source function was negative during the entire measurement period. When take an average value along the 3.1km long Colwyn Bay coastline, the space integrated source function gives 0.315 m/year beach recession. The average rate of beach recession along the Colwyn Bay, calculated directly from historic shoreline survey measurements is 0.69 m/year. This means that the residual wave effects and all other morphodynamic drivers such as tides and wave induced currents which are included in the source function contributes to around 50% of sediment transport. The remainder may be attributed to net long term wave effect.

5. Conclusions

The research presented in this paper uses an advection-diffusion type formulation to describe beach plan shape change. In the advection-diffusion formulation, shoreline change from long-term net effect of incident waves is modelled through a longshore-varying diffusion coefficient. Contributions from residual wave effects, tides and all other anthropogenic drivers are modelled through a time and space-varying source function. Both the diffusion coefficient and the source function are site-specific unknowns and need to be determined from historic measurements of the beach concerned.

The methodology used in this paper can be used to model shoreline change at a range of time scales. However, the level of details that could be modelled at a given site at a given time scale may vary on the precision, frequency and length of historic data available on the site climate of the chosen site. The implementation of the method is straightforward and very computationally efficient. Although the focus of this paper is to demonstrate the modelling approach and assess beach plan shape change, the method can, in principle, be used to predict future shoreline change given that past site climate would not significantly change in future. Thus, given $\overline{K}(x)$ and suitable parameterisations or forecasts of $G(x,t)$, Equation (4) can be solved forward to predict future shoreline change.

Here, the time-mean diffusion coefficient represents the net effect of incident waves thus relating it to longshore transport contribution to shoreline change. Even though the source function includes all effects other than the net effects of incident waves, the strong seasonal signature seen in it indicates that it predominantly contains the effect of the fluctuation of incident wave field around the mean. Therefore, the source function mainly represents the cross-shore transport contribution to residual shoreline change at shorter time scales. From the diffusion coefficient and the source function derived for Colwyn Bay, it is clear that the beach plan shape change at the west side of the bay is predominantly governed by short term beach variations while the east side is governed by long term variations. A notable feature of the method is its ability to distinguish cross-shore and longshore transport contributions to shoreline change at different time scales.

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